Date: 01/10/2017 Deadline: Wednesday (04/10/2017) Morning *Homework 1 for PhD Scholars, 1st semester.*

1. From the book, read about geometric random variable. Write its pmf, find its mean, variance. Also find its moment generating function. Using MGF, find its mean, second moment and hence variance. Verify that you get same result that you get via usual method.

2. Let X have a Geometric distribution with probability of success as p. For $n \geq 0$, show that

$$P(X > n) = (1 - p)^n \tag{1}$$

3. You toss n coins, each showing heads with probability p, independently of the other tosses. Each coin that shows tails is tossed again (coin is tossed only twice). Let X be the total number of heads.
   a. What type of distribution does X have? Specify its parameter(s).
   b. What is the probability mass function of the total number of heads X?

4. Let the function $p_X(x)$ be uniform pmf, defined as

$$p_X(x) = \begin{cases} \frac{1}{b-a-1}, & \text{if } x = a, a+1, \dots, b \\ 0, & \text{otherwise} \end{cases}$$

   Find mean and variance of this discrete random variable.

5. Let X and Y be random variables such that $\mathsf{E}[X] = 2$, $\mathsf{E}[Y] = 3$, and $\mathsf{Var}(X) = 4$.

   a. Show that $\mathsf{E}(X^2) = 8$.
   b. Determine the expectation of $2X^2 + Y$.

6. Consider the variables X and Y with joint probability density

$$f(x, y) = \begin{cases} \frac{2}{75}(2x^2y + xy^2) \text{ for } 0 \leq x \leq 3 \text{ and } 1 \leq y \leq 2 \\ 0, \text{ otherwise} \end{cases}$$

   a. Find marginal densities $f(x)$ and $f(y)$.

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   b. Compute $\mathsf{E}[X]$, $\mathsf{E}[Y]$, and $\mathsf{E}[X+Y]$.

   c. Compute $\mathsf{E}[X^2]$, $\mathsf{E}[Y^2]$, $\mathsf{E}[XY]$, and $\mathsf{E}[(X+Y)^2]$,

   c. Compute $Var(X+Y), Var(X)$, and $Var(Y)$ and check that $Var(X+Y) \neq Var(X) + Var(Y)$.

   d. Compute $Cov(X,Y)$, and also compute correlation coefficient ρ_{XY}.

7. Let a continuous random variable X be given that takes values in $[0,1]$, and whose distribution function F satisfies $F(x) = 2x^2 - x^4$ for $0 \leq x \leq 1$.

   a. Compute $P(1/2 \leq X \leq 3/4)$.

   b. What is the probability density function of X?

8. Learn about the following continuous random variables and find mean, variance of each of them. Show all the steps: 1) Uniform random variable over $[a,b]$, $a,\ b \in \mathrm{R}$ (2) Pareto random variable